Chapter 0

Mathematical Notions

sets, sequences, tuples, functions, relations, strings, and languages

sets

Set is a group of objects represente as a unit.

sets may be described formally

$A = \{ x \mid x \in \text{Integers and } x > 0 \text{ and } x < 4 \}$

$A = \{1, 2, 3\}$

empty set $\{\} = \emptyset$

union

intersection

complement

$B = \{a, b, c\}$

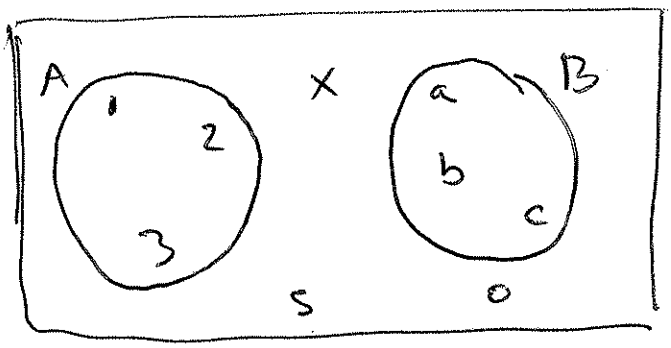
$A \cup B = \{1, 2, 3, a, b, c\}$

$A \cap B = \emptyset$

$\overline{A}$ complement of A

elements that are not in A

Venn diagram



$\bar{A} = \{ x, s, a, b, c, o \}$

Sequence

It is a list of objects in some order.

$A = \{1, 2, 3\}$

$A = \{2, 1, 3\}$

123 $(1,2,3) \neq (3,2,1)$

sequences that are finite are called tuples

tuples with n elements are called n-tuple

Power set of A is the set of all subsets of A

$A = \{1, 2, 3\}$

power set of $A = \{ \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}, \emptyset \}$

length of set $A = |A|$

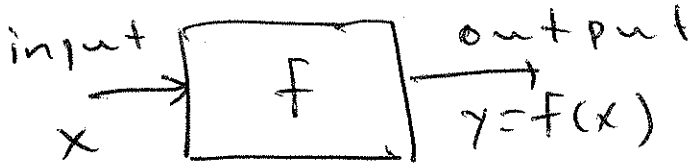
length of power set of $A = 2^{|A|}$

Cartesian Product (Cross product)

$$A \times B = \{(1,a),(1,b),(1,c),(2,a),(2,b),(2,c),(3,a),(3,b),(3,c)\}$$

$$|A \times B| = |A| \cdot |B|$$

Functions



function is an object that sets up an input-output relationship.

$$f(x) = 3x^2 + 5x - 4$$

$$f(0) = -4$$

$$f(1) = 3 + 5 - 4 = 4$$

domain D

range R

$f: D \to R$

$\{1,2,3\} \to \{a,b,c\}$

$f(1) = b$

x	f(x)
1	b
2	a
3	c

$\{1,2,3,4,5\} \rightarrow \{a,s\}$

x	f(x)
1	a
2	s
3	a
4	s
5	a

predicate (property) is a function range is {TRUE, FALSE}

Relation is a property whose domain is a set of k-tuples

mother (rose, john)

Functions and Relations

onto

A function that does use all the elements of the range.

Example

$f: \{1,2,3,4\} \rightarrow \{1,2,3,4\}$

n	f(n)
1	2
2	3
3	2
4	3

Is f onto? NO

Arity of the function is the number of arguments

arguments	type of fuction
1	unary
2	binary
k	k - ary

Equivalence relation

A binary relation that contains elements that have the same property.

An equivalence relation has the following 3 conditions:

(1) Reflexive, R is reflexive if
$\forall x, \; x R x \quad R(x,x)$

(2) Symmetric, R is symmetric if
$\forall x, y \quad x R y \Rightarrow y R x$

(3) transitive, R is transitive if
$\forall x, y, z \quad x R y, \; y R z \Rightarrow x R z$

Mother is not an equivalence relation

Beats (page 9) is not an equivalence relation.

Example 0.11 (page 9)

Define an equivalence relation on the natural numbers

$i \equiv_7 (x,y)$ true if $x-y$ is a multiple of 7

$$i\equiv_7 = \{(14,7), (42,7), (8,1), (15,1), (10,3), \ldots\}$$

$$i\equiv_7 = \{(x,y) \mid x \in N, y \in N, x-y \text{ is a multiple of } 7\}$$

(1) Is $i\equiv_7$ Reflexive? yes

$i\equiv_7 (x,x)$ $\quad x-x = 0$

0 is a multiple of 7.

(2) Is $i\equiv_7$ symmetric? yes

$i\equiv_7 (x,y) = x-y = 7\cdot k$

$i\equiv_7 (y-x) = y-x = -7k$

$-7k$ is a multiple of 7

(3) Is $i\equiv_7$ transitive? yes

Known

$i\equiv_7 (x,y) = x-y = 7\cdot k_1$

$i\equiv_7 (y,z) = y-z = 7\cdot k_2$

$i\equiv_7 (x,z) = x-z \stackrel{?}{=} 7\cdot k_3$

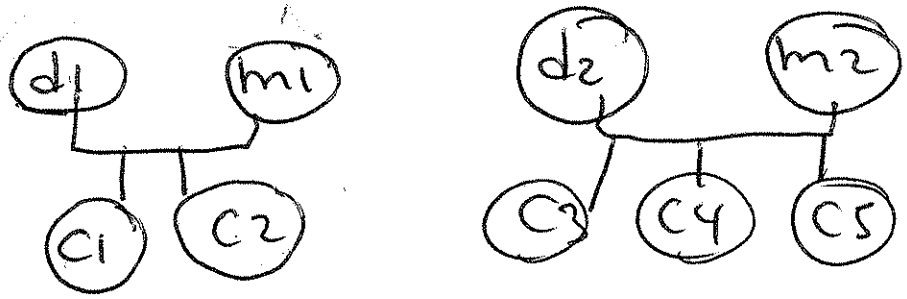
$(x-y)+(y-z) = x-z$

$7k_1 + 7k_2 = 7(k_1+k_2)$

$7(k_1+k)$ is a multiple of 7

$\equiv_7$ is an equivalence relation

the following family trees define a predicate "related"



related (d1, c3) = false
related (c3, c5) = true
related (d1, m1) = true

Find a possible grouping of tuples to make an equivalence relation.

Equivalence relation would be one family.

strings and languages

Alphabet is a non empty finite set. Σ, T

$\Sigma = \{0, 1\}$

symbols are the members of the alphabet.

strings over an alphabet

is a finite sequence of symbols from a given alphabet

$\Sigma = \{0, 1\}$

strings 01, 101, 0010,

$\Gamma = \{a, b, c, d, e\}$

strings cad, dea, cab, caaba, ...