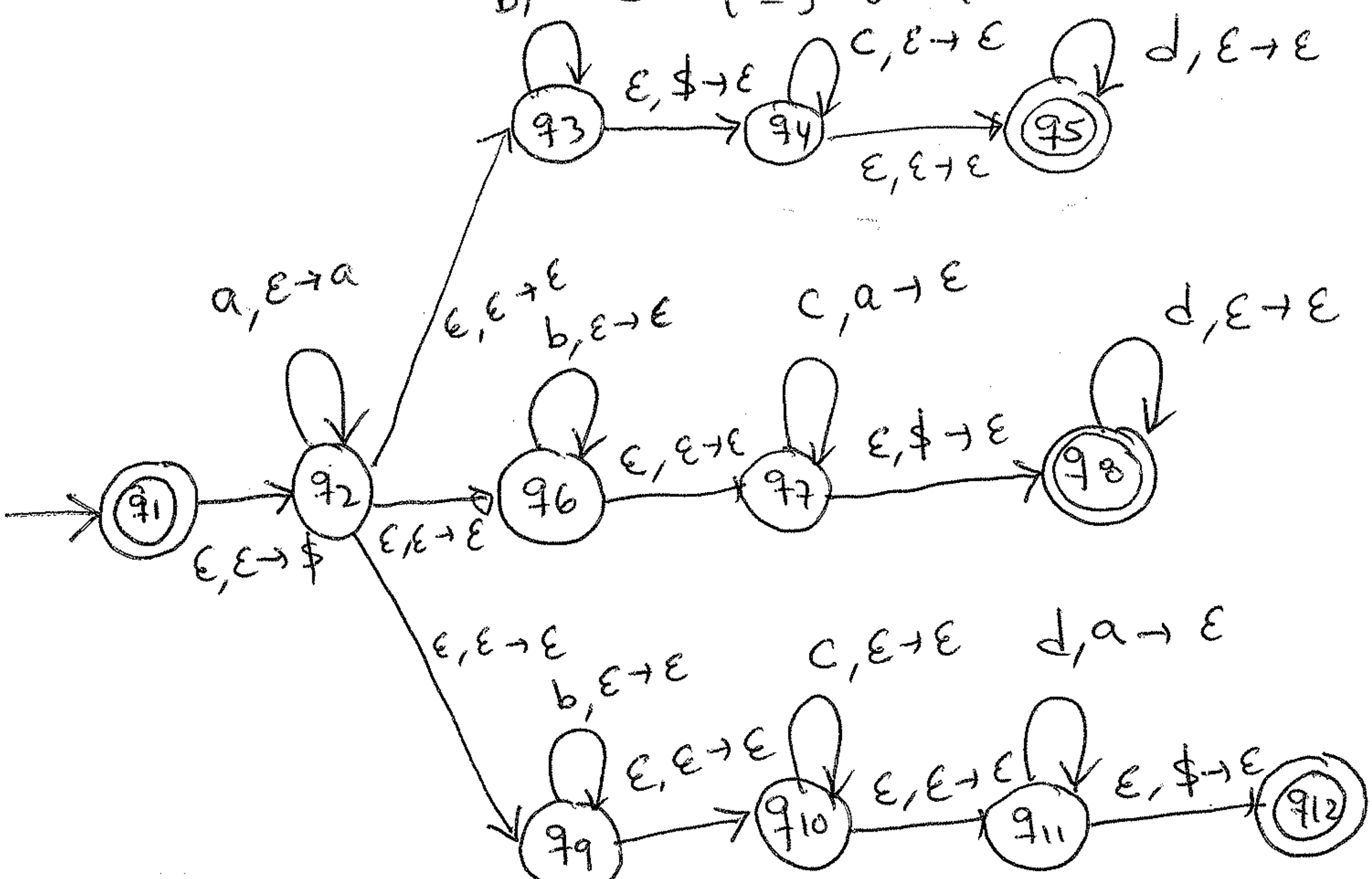
Design a PDA that accepts strings from L2

$L2 = \{a^i b^j c^k d^l \mid i, j, k, l \geq 0$ and $i = j$ or $i = k$ or $i = l\}$

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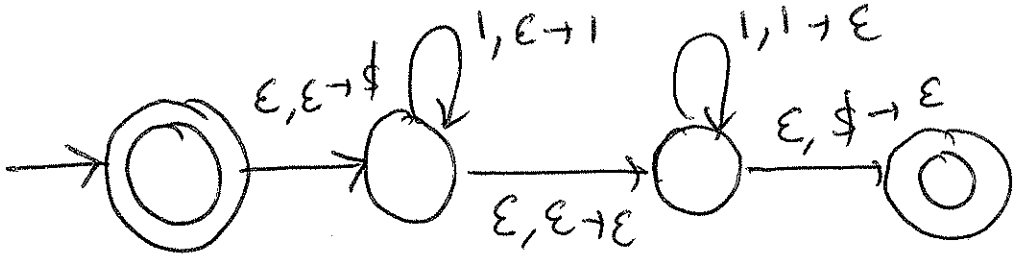
Design a PDA for $L1 = \{a^n b^n c^n \mid n \geq 0\}$

This is not possible because L1 is not a CFL

Find a PDA for $L3 = \{ 1^{2n} \mid n \geq 0 \}$

$$1^{2n} = 1^n 1^n$$




Exercise 2.9 page 129

Give a CFG that generates

$A = \{ a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0 \}$

$S \to XC \mid AY$

$X \to aXb \mid \varepsilon$

$C \to cC \mid \varepsilon$

$A \to aA \mid \varepsilon$

$Y \to bYc \mid \varepsilon$

X matching a's with b's

C ignoring c's

A ignoring a's

Y matching b's with c's

Exercise 2.10 page 129
Informal description of PDA for exercise 2.9

1. Nonderministically branch to either stage 2 or stage 5
2. Read and push a's
3. Read b's while popping a's
4. If b's finish when stack is empty, skip c's on input and accept; else reject
5. Skip a's on input
6. Read and push b's
7. Read c's while popping b's
8. If c's finish when stack is empty, accept; else reject.

Example Get PDA for L

$L = \{ w w^R \mid w \in \{0,1\}^* \}$

if $w = 110$
$w^R = 011$
$110011$


See figure 2.19 page 114

# Equivalence of PDAs with CFGs

**Theorem 2.20** A language is CF iff some PDA recognizes it.

**Lemma 2.21** If a language is CF, then some PDA recognizes it.

**Lemma 2.22** If some PDA recognizes a language, then the language is CF.

## Informal description of PDA P

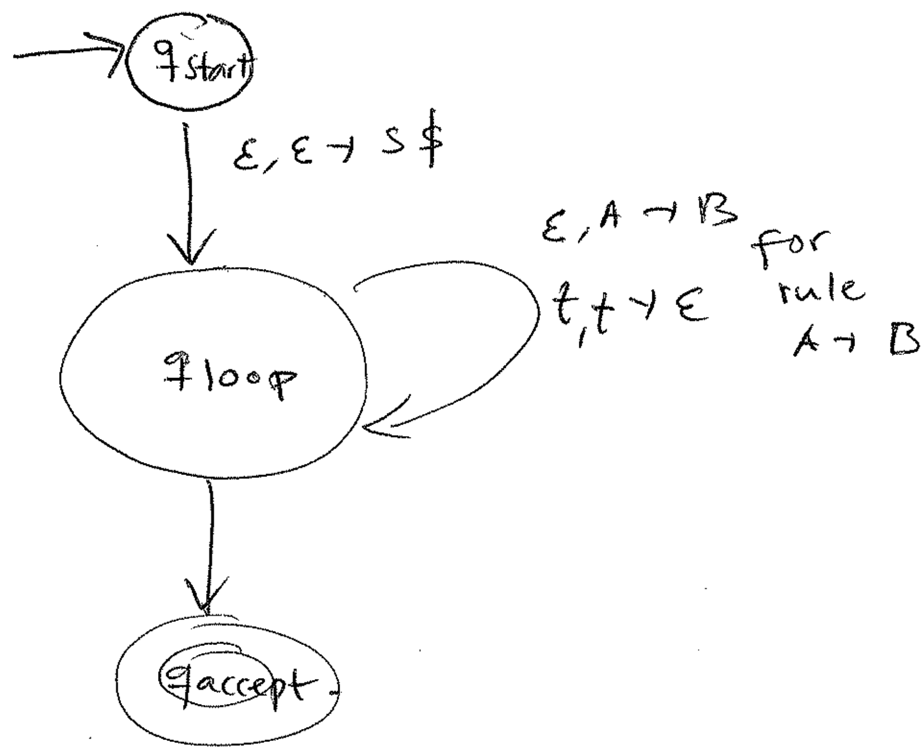
1. Place a marker symbol $ on the stack and then the start variable of the CFG

2. Repeat
   (a) If the top of the stack is a variable symbol A, select one of the subrules for A and substitute the rhs of it for A.

right hand side

(b) If the top of the stack is a terminal symbol, t, read the next symbol from the input and compare it to t. If they match, continue, otherwise this PDA fails.

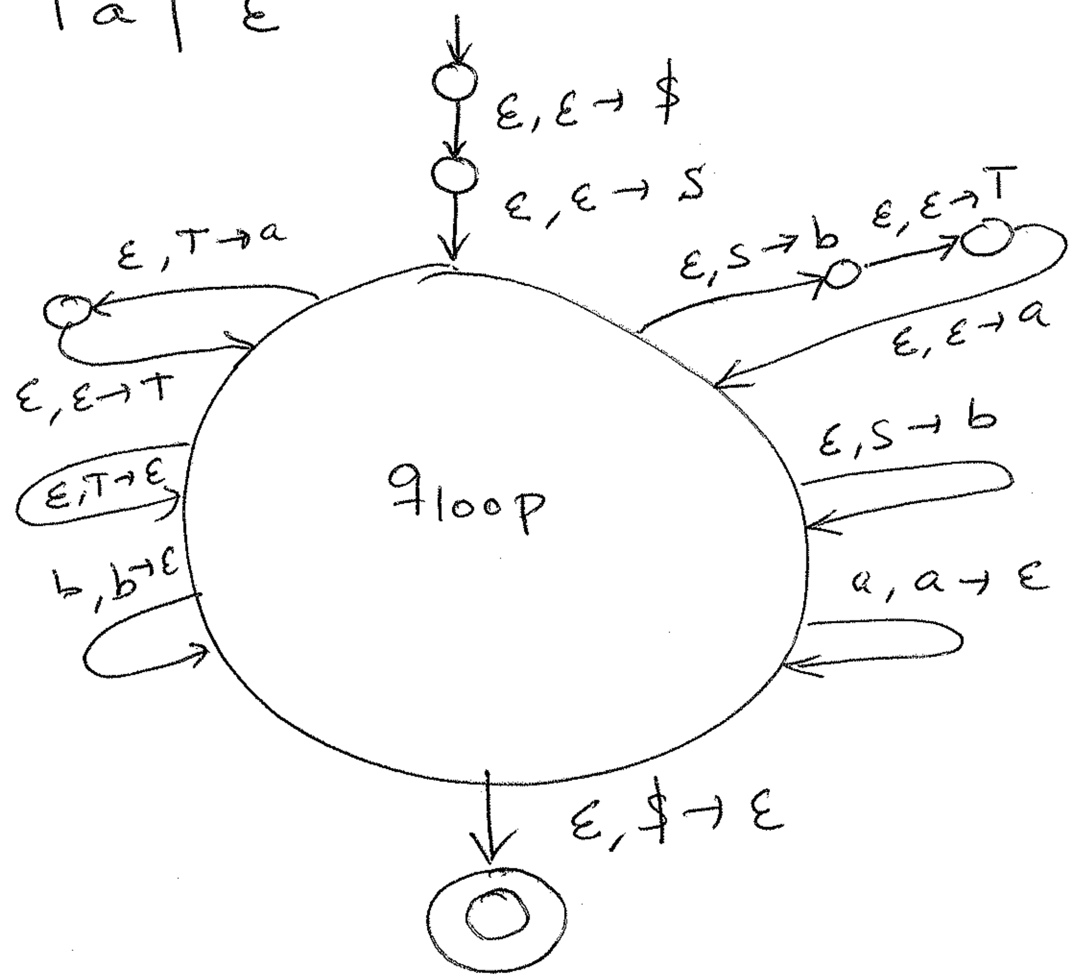
(c) If the top of the stack is the marker ($), enter the acept state. Accept the input string if all the input has been read.



Example 2.25 Construct a PDA $P$ for the following CFG $G$:

$S \to aTb \mid b$

$T \to Ta \mid \varepsilon$

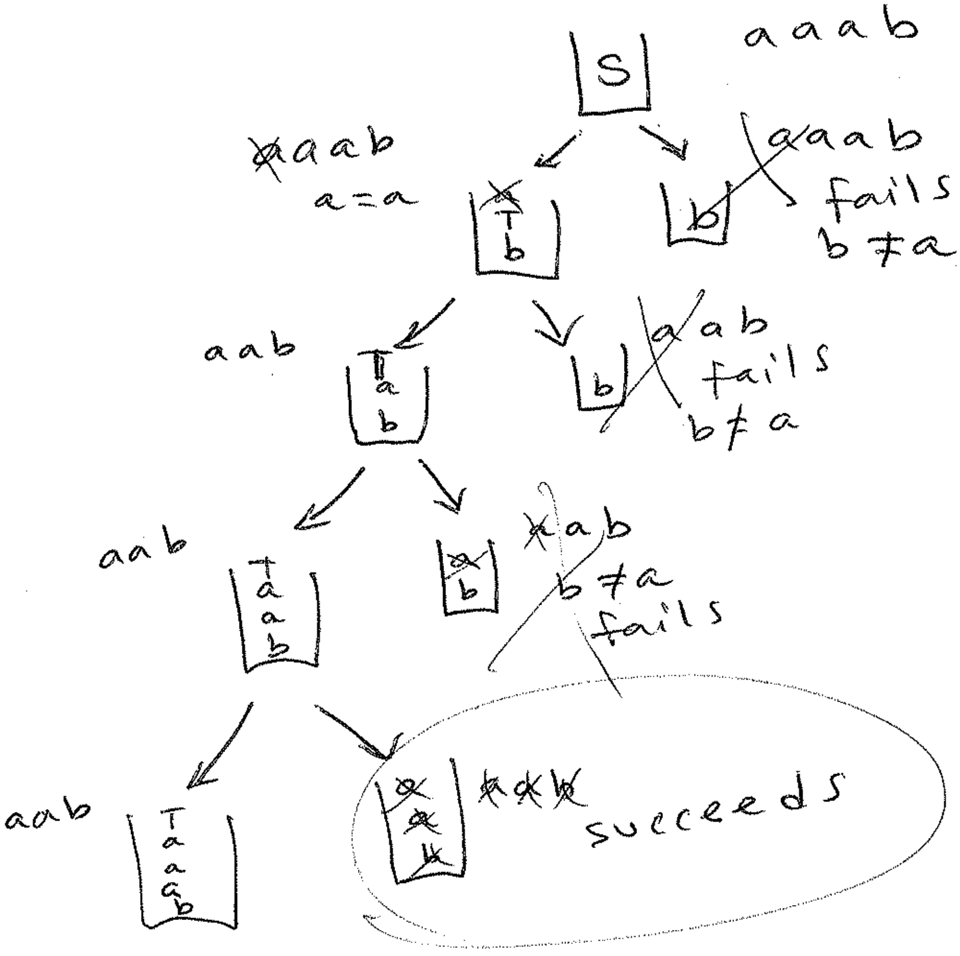


Describe the language accepted by the previous PDA P, and show how P processes the string aaab

L(G) is any number of a's followed by one b

$L(G) = \{ b, ab, aab, aaab, a\ldots ab, \ldots \}$

$a^* b$



Are regular languages also CFL?

Yes

Every regular language is recognized by a FA, and all FAs are PDAs that ignore their stack. Therefore, all regular languages are also CFLs.


Remore $\varepsilon \rightarrow$ productions from the following and then convert into equivalent CNF

$A \rightarrow aB \mid bD$

$B \rightarrow aC \mid bC$

$C \rightarrow aC \mid bC \mid \varepsilon$ $\qquad C \rightarrow \varepsilon$

$D \rightarrow aB \mid bC$

---

$A \rightarrow aB \mid bD$

$B \rightarrow aC \mid bC \mid a \mid b$

$C \rightarrow aC \mid bC \mid a \mid b$

$D \rightarrow aB \mid bC \mid b$

$A \to XB \mid YD$
$B \to XC \mid YC \mid a \mid b$
$C \to XC \mid YC \mid a \mid b$
$D \to XB \mid YC \mid b$

$X \to a$
$Y \to b$

## Non-context-free languages

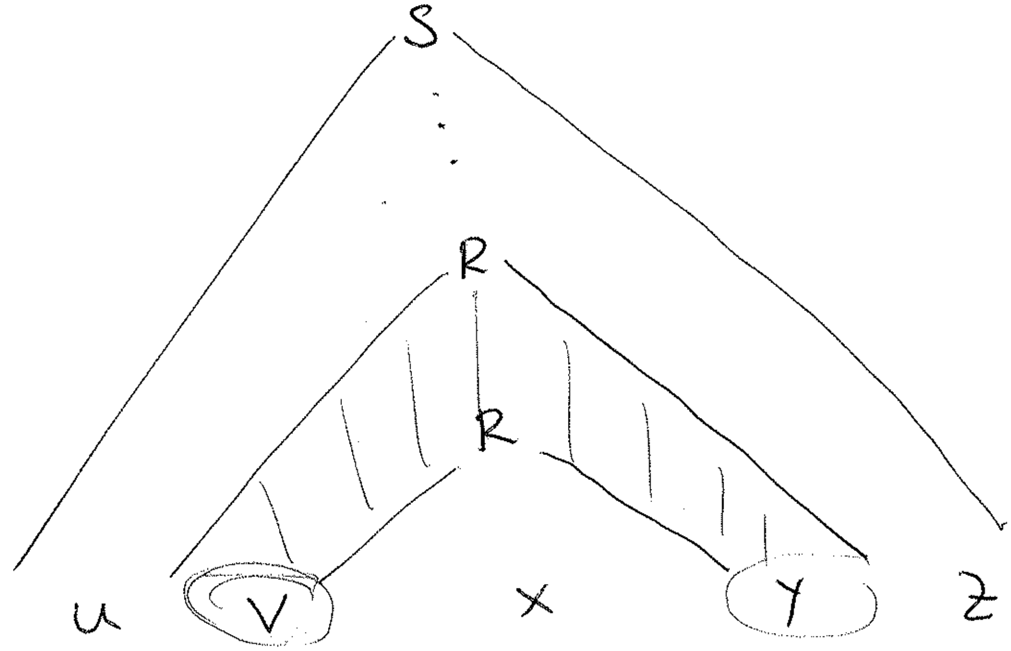
**Theorem 2.34** Pumping Lemma for CFLs

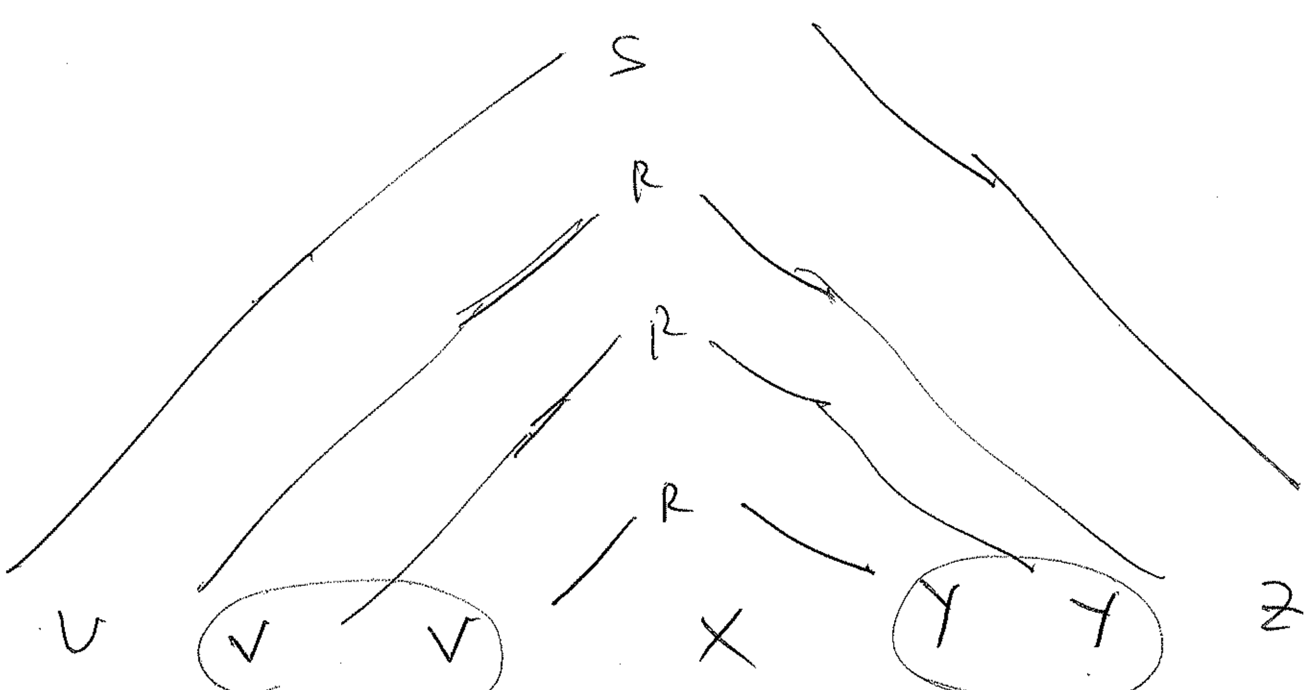
If $A$ is a CFL, then there is a number $p$ (the pumping length) where, if $s$ is any string in $A$ of at least length $p$, then $s$ may be divided into 5 parts $s = uvxyz$ satisfying the conditions

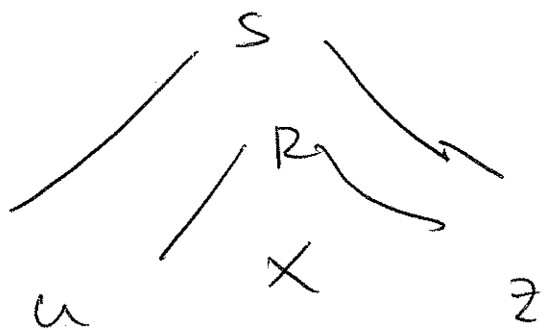
1. For each $i \geq 0$, $uv^ixy^iz \in A$
2. $|vy| > 0$
3. $|vxy| \leq p$

$P \simeq |V|$

$|s| > P$




S
R
u
x
z

Example 2.39 Show that $B = \{a^n b^n c^n / n \geq 0\}$ is not a CFL

1. Assume that B is CFL
2. Let P be the pumping length that is defined by the pumping lemma
3. Let $S = a^p b^p c^p$ $|S| = 3P$
4. S may be divided into five parts $\forall i \geq 0$ $S \in B$

   $S = u v^i x y^i z$

   $|vy| > 0$ $|vxy| \leq P$

if $u = \varepsilon$ when $i > 1$

S will have more a's than b's and c's

5. Therefore, $S \notin B$ for $i > 1$

   then B is not a CFL

Exercise 2.2 (a) page 128

Use the languages $A = \{a^m b^n c^n / m, n \geq 0\}$ and $B = \{a^n b^n c^m \mid m, n \geq 0\}$ together with the previous example 2.39 to show that the class of CFL is not closed under intersection.

Is A CFL ? yes

Is B CFL ? yes

$A \cap B$ ? CFL ?

$$A \cap B = \{a^n b^n c^n \mid n \geq 0\}$$

the intersection of A and B gives a non CFL. Therefore, CFL are not closed under the intersection.