The halting Problem

$A_{TM} = \{ \langle M, w \rangle \mid M \text{ is a TM that accepts } w \}$

Show that the set of real numbers is uncountable

n	f(n)
1	1.1
2	0.03
3	2.54
4	3.3333
5	25
6	40
⋮	

0.2135

there is always a real number not paired with a natural number. Therefore, there is no correspondence between the Natural ~~real~~ numbers and the set of irrational (real) numbers. Then the set of real numbers is uncountable.

Show that the set of all strings over alphabet $\Sigma = \{0,1\}$ are countable for any Σ

(show Σ^* is countable)

Σ^*	ε	0	1	00	01	10	11	100	101
N	1	2	3	4	5	6	7	8	9

(A) Show that the language of strings that represent Turing Machines is countable

(B) Show that the set of infinite binary sequences B is uncountable

(A) The set of all Turing Machines is countable because each Turing Machine M has an ~~e~~coding into a string $\langle M \rangle$ and the set of strings is countable.

Σ^*	ε	0	1	00	01	10	11	100	101

1	0 1 0 0 0 0 1 ...
2	1 0 1 0 1 1 0 ...
3	1 0 1 1 1 0 0 ...
4	1 0 0 0 1 ~~1~~ 0 ...
⋮	

$X = 1101 \cdots$

Show that the set of all languages L over alphabet Σ is uncountable.

characteristic sequence

Σ^*	ε	0	1	00	01	10	11	000	001	...
$L1 =$	1	1	1	0	0	0	0	0		...
$L2 =$	1	0	1	0	1	0	0	1		...

Each language is associated with a characteristic sequence which is an infinite binary sequence. these sequence are uncountable; hence, the set of all languages is uncountable.

Show that A_{TM} is undecidable

$A_{TM} = \{\langle M, w\rangle \mid M$ is TM and M accepts $w\}$

H is a decider of A_{TM}

$$H(\langle M, w\rangle) = \begin{cases} \text{accept}; & \text{if } M \text{ accepts } w \\ \text{reject}; & \text{if } M \text{ does not accept } w \end{cases}$$

construct a new TM D that uses H as a subroutine

D calls H to determine what M does when its input ~~#~~ is $\langle M\rangle$

if M accepts $\langle M\rangle$ then D rejects;

if M rejects $\langle M\rangle$ then D accepts.

D = "On input $\langle M\rangle$ where M is TM

1. Run H on input $\langle M, \langle M\rangle\rangle$
2. Output the opposite of what H outputs

$$D(\langle M \rangle) = \begin{cases} \text{accept, if } M \text{ does not accept } \langle M \rangle \\ \text{reject, if } M \text{ accept } \langle M \rangle \end{cases}$$

What happens when we execute D on $\langle D \rangle$?

$$D(\langle D \rangle) = \begin{cases} \text{accept, if } D \text{ does not accept } \langle D \rangle \\ \text{reject, if } D \text{ does not reject} \end{cases}$$

this is a contradiction and neither D nor H do exist.

A Turing - unrecognizable language

A language is decidable iff both it and its complement are Turing-recognizable.

If A is decidable then $\overline{A}$ is decidable

then A and $\overline{A}$ are Turing recognizable

Let M_1 be the recognizer of A
Let M_2 be the recognizer of $\overline{A}$

$M =$ " On input w

1. Run both M_1 and M_2 on input w in parallel
2. If M_1 accepts, accept.
   If M_2 accepts, reject."

Corollary 4.23

$\overline{A_{TM}}$ is not turing-recognizable

A_{TM} is not a decider

the $\overline{A_{TM}}$ is not turing-recognizable otherwise A_{TM} would be decidable.

Exercise 4.7

Let $T = \{(i, j, k) \mid i, j, k \in N\}$

Show that T is countable

$$f(i, j, k) = 2^i 3^j 5^k$$

Exercise 4.5

$$X = \{1, 2, 3, 4, 5\}$$

$$Y = \{6, 7, 8, 9, 10\}$$

$f: X \to Y$

n	$f(n)$
1	6
	7
2	6
3	
4	7
5	6

$g: X \to Y$

n	$g(n)$
1	10
2	9
3	8
4	7
5	6

(a) Is f one-to-~~one~~one? NO problem $f(1) = f(3)$

(b) Is f onto? NO

(c) Is f a correspondence? NO

(d) Is g one-to-one? yes

(e) Is g onto? yes

(f) Is g a correspondence? yes

Are N and $E \cup \{x\}$ the same size? where N is the set of Natural Numbers and E is the set of positive even numbers

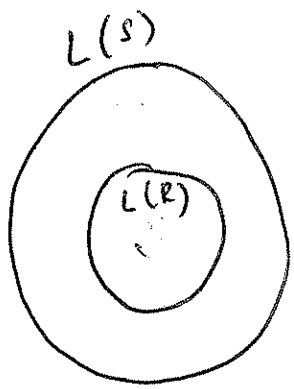
$$f(n) \begin{cases} x ; & \text{if } n=1 \\ 2(n-1) & \text{otherwise} \end{cases}$$

there is a correspondence between N and $E \cup \{x\}$. Therefore, they are the same size.

Exercise 4.12 page 183

Let $A = \{ \langle R, S \rangle \mid R$ and S are regular expressions and $L(R) \subseteq L(S) \}$. Show that A is decidable.

Use indirect method to prove it.



$$\overline{L(S)} \cap L(R) = \emptyset$$

$X =$ "On input $\langle R, S \rangle$ where R and S are regular expressions

1. Construct DFA E such that
   $L(E) = \overline{L(S)} \cap L(R)$
   using procedures from chapter 1
2. Run TM T on $\langle E \rangle$
3. If T accepts, accept; otherwise reject ".

Exercise 4.15

Let $A = \{ \langle R \rangle \mid R$ is a regular expression describing a language containing at least one string that has 111 as a substring $\}$
($w = x\,111\,y$ $\exists$ x and y)
Show that A is decidable.
Use Indirect method.

$X1 =$ "On input $\langle R \rangle$ where R is a regular expression

1. Construct DFA E that accepts $\Sigma^* 111 \Sigma^*$ using procedures from chapter 1

(2) construct DFA B such that
$L(B) = L(R) \cap L(E)$

(3) Run TM T on B

(4) If T accepts, reject.
otherwise, accept."

Computational Models

Final Exam Study Guide

General Information

Date: Monday December 19, 2011
Time: 5:45 pm - 7:45 pm (check on Solar)
Location: Regular classroom

Exam is cummulative and covers all chapters discussed in class.

Topics may include all or some of the following:

1. Proofs by induction, contradiction or construction
2. Functions and relations
3. Formal descriptions of languages generated by CFG, RE, PDA, TM, FA, etc.
4. Converting NFAs to DFAs
5. Define CFG from NFAs or DFAs
6. Pumping lemma for either RL or CFL

7. Define regular expressions for languages generated by CFG, RE, FA, PDA, etc.

8. Converting Grammar into CNF

9. Removing useless symbols, unit production or ϵ-productions

10. Complement of a NFA

11. Identify minimum pumping length as shown in exercise 1.55 page 91

12. Give CFG that generates a given language

13. Implementation level description of TM

14. Proof of decidability

15. the halting problem

Exercise 1.55 page 91

a. 0001^* 4

b. 0^*1^* 2

c. $001 \cup 0^*1^*$ 2 j. Σ^* 1

i. 1011 5

f. ϵ 1